2010 LICB International Econometric M&R Cost Benchmarking of Network Rail (2008 UIC dataset update)

Technical Support Paper

Office of Rail Regulation

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1. Introduction and summary of models

This publication documents the updated work carried out in 2010 using the most recent UIC/LICB dataset (up to and including the financial year 2008; UK 2008/09 financial year for Network Rail). The purpose of this document is to explain how ORR arrived at a preferred estimated model and to demonstrate the process by which this model has been selected for this updated analysis.

This analysis is the first official post-Periodic Review 2008 (PR08) update and follows from work undertaken with ITS Leeds and shared with Network Rail (NR) and UIC in 2010. An earlier version of this update has been presented to UIC/LICB in March 2010. It should be noted that the update work is based largely on applying the same methods adopted in PR08 to an updated dataset. We will examine further, potential developments to the methodology in subsequent work.

We ran a number of models, with sensitivities for different outputs and network characteristics. We started with a full translog specification estimated as a cost frontier (using stochastic frontier analysis, which makes use of maximum likelihood estimation techniques), both with outputs/density and network characteristics at first and second orders. However, this model did not converge, probably due to its complexity (translog models contain many variables).

In order to explore the translog functional form further, we estimated the same model using ordinary least squares, which does not assume the existence of a best practice frontier to encompass the data. Furthermore, this approach does not recognise the panel structure of the data (it treats each observation as an independent firm, rather than recognising that the data consists of a number of firms, with multiple-years observations for each firm). If the assumption that the omitted, firm-specific effects are uncorrelated with the regressors is true (as is assumed in the other maximum likelihood stochastic frontier models that we estimate), then this approach should give unbiased parameter estimates.

This approach produced implausible results both in terms of estimated coefficients and cost/output elasticities (Model 1A). We therefore restricted Model 1A to eliminate insignificant variables (Model 1A1). This model still produced unsatisfactory estimates, given the negative (though insignificant) freight density cost elasticity at the sample mean (a counter-intuitive result). The negative and significant coefficient on the proportion of electrified track might also be considered counter-intuitive. Looking at the elasticities away from the sample mean we see implausibly large negative elasticities.

We therefore utilised a Cobb-Douglas cost function, as in PR08 (Model 1B) - making use of the time varying inefficiency stochastic frontier model specification (Cuesta, 2000). This model allows inefficiency to vary over time, but in a structured way.

Model 1B (Cobb-Douglas) includes all traffic, scale and network characteristics variables together, resulting in the passenger train density cost elasticity being negative (albeit statistically insignificant), which is not in line with theoretical expectations. We thus proceeded to drop statistically insignificant explanatory variables, dropping the ones showing the lowest t-statistic (in absolute value) first. This leads us to a series of refined Cobb-Douglas specifications, namely Models 1B1 to 1B3. These model are variants on Model 1B, retaining the Cobb-Douglas specification under Cuesta (2000) as in PR08.

Out of these variants on the main theme, we preferred Model 1B3. In this model, the overall variation in cost with respect to traffic is in line with estimates from previous studies, though the estimate for freight is very close to zero and is statistically insignificant (which could be the result of multi-collinearity). However, it does have the advantage of allowing for different elasticities for different types of traffic. It is therefore our preferred model, and will be subject to a further variant in terms of Cuesta (2000) error term specification, as discussed later. It should be noted that the model chosen gives the most favourable score for Network Rail as compared to the other possibilities (1B, 1B1 and 1B2), and is thus conservative.

In order to investigate whether the advantages of a more flexible functional form could be obtained within this modelling framework, we expanded Model 1B3 by adding second order terms (though as noted we had to place some restrictions on the model in order for the model to estimate). We find that the Cobb-Douglas restriction (including a time-trend squared variable) cannot be rejected based on a Likelihood Ratio (LR) test against the more general functional form. For this reason, we still prefer Model 1B3, the Cobb-Douglas specification. The TL model implies an efficiency score of 0.76 for Network Rail. However, again this model has a negative freight elasticity at the sample mean that is close to being statistically significant (p value of 0.11), and many of the elasticities away from the sample mean are therefore negative. Taking all of these factors into account, we therefore prefer the Cobb-Douglas specification in 1B3.

We also made some preliminary investigation into modification to the Cuesta (2000) time varying inefficiency specification adopted. The results for Network Rail do not appear to be much affected, but further work will continue in this area going forward. We also tested an alternative model specification with the main output variables specified as total train density (passenger and freight train density added together); labelled Model 1D. This model produces similar scores for Network Rail to 1B3 (0.63 in 2008).

We then estimated the same models, but taking track-km as the scale variable (as opposed to route km[1]). We also tried models with the network characteristics variables as natural values as opposed

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[1] It could be argued that route-km is the preferred scale indicator, similar to line length in utility industries, as it measures the distance covered by the network. Our modelling work also provided a weak

to logs. This gave rise to a family of models which we labelled Model 2, with variants. Results in terms of efficiency scores are not different from the main family of models (all in natural logarithms), however individual estimated coefficients are less plausible. For this reason, we dropped the “Model 2” family. We note that some of these differently scaled models, especially 2C and 2D as described in the main text below, produced lower scores for Network Rail than those resulting from the main “Model 1” family.

Models 3A, 3B are finally a variant on the main preferred model, testing for more restrictive assumptions concerning efficiency variation over time. Model 3A is the Battese and Coelli (1992) model, of which Cuesta (2000) is a general case. 3B is the Pitt and Lee (1981) model, which again is nested within both Battese and Coelli (1992) and Cuesta (2000). In Model 3A, the direction of inefficiency change is restricted to be the same for all firms, which is unrealistic. In Model 3B, inefficiency is assumed constant for all firms over time, which is again unrealistic. We tested both restrictions against our preferred model (Cuesta, 2000), and we rejected both of these restrictions. Therefore, we proceeded with the Cuesta (2000) error specification, as in PR08.

A final cross-check concerns the steady state adjustment. We performed all of the above work keeping the steady state adjustment on renewal costs (2.5% renewal rate per annum) in place, exactly as in PR08. Then we removed the steady state adjustment for our preferred model, leading to a score of 0.53 for Network Rail in 2008. We tried also with a different formulation of the frontier, which is generally harsher (Corrected Ordinary Least Squares, COLS), obtaining a score of 0.65[2] for Network Rail in 2008. This latter check was only performed for the sake of completeness. ORR does not intend to replace stochastic frontier analysis in PR13. ORR might, however, review its approach to the steady state adjustment, and this is currently being discussed with Network Rail.

2. Starting point: full Trans-logarithmic (Translog, TL) model

In order to avoid cost allocation issues between maintenance and renewal activities, the dependent variable in all of our models is total maintenance and renewal cost combined. ORR has reviewed regulatory practice and considers this “total cost” approach to be now traditional in best European and Australian regulatory practice (electricity, gas, telecommunications). It has been examined for future usage by Ofgem (electricity and gas distribution/transmission) in its recent RPI-X@20 project.

However, when total cost analysis is used, capital measurement is not straightforward. Accounting based measures are problematic given the long asset lives in railways and different depreciation and revaluation policies across countries. On the other hand, capital expenditure cash based measures could fluctuate from year to year for reasons other than changes in efficiency. One way round this is to average capital expenditure data over a number of years, though this reduces the number of observations for analysis. In this analysis we have chosen to adjust Network Rail’s cost data to reduce costs (increase costs) when renewal activity is considered to be above (below) steady-state.

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preference for the use of route-km on statistical grounds. ORR’s work also shows that this has been demonstrated in other network industries.

[2] This score is quoted against the upper quartile, in line with the approach used by other economic regulators.

We recognise that there is insufficient hard data to make similar adjustments for other railways, and ideally such adjustments would be made. However, we consider that our approach is robust for the following reasons. First, ORR's inspection of the data and anecdotal evidence did not give any reason to believe that the frontier firms - which are of particular importance in determining the position and shape of the frontier - are substantially away from steady-state. Second, the stochastic frontier approach itself (which seeks to separate noise and inefficiency), and the use of panel data over an 11-year period, provide further safeguards against the risk of misinterpreting low costs in one particular year as evidence of efficient operation, and thus creating an unrealistic benchmark. Third, as final checks on the modelling work, the results of the preferred model are compared against a model that does not include any adjustment to Network Rail's raw cost data.

We use a general to specific methodology starting with a "non-parsimonious" model containing all of the following variables (Model 1; all in natural logarithms); variable names in parentheses:

- Route-km (ROUTE);
- % of single track (SING);
- % of electrified track (ELEC);
- Passenger train density per route-km (PASSDR);
- Freight train density per route-km (FRDR);
- Or total train density per route-km (TOTDR);
- Station density per route-km (STAT); and
- Switch density per track-km (SWITCH).

We started by testing a full translog specification, estimated using the flexible "Cuesta 2000" model[3] as per PR08.

If we include all first and second order (squared and interaction) terms for the above variables, then the Cuesta model, which is based on a maximisation procedure (maximum likelihood estimation, MLE) fails to converge (i.e. to find an optimising solution). In order to investigate the translog form as compared with the Cobb-Douglas form, we thus proceed with ordinary least squares (OLS).

It has been pointed out, both during PR08 and afterwards, that individual countries in the LICB dataset (as described in the PR08 econometric reports[4]) might feature unobserved effects, either controllable or uncontrollable by railway managers, which may affect our estimates of inefficiency.

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[3] Cuesta, R.A. (2000), A Production Model With Firm-Specific Temporal Variation in Technical Inefficiency: With Application to Spanish Dairy Farms. *Journal of Productivity Analysis 13:2*, 139-158.

[4] Please see: http://www.rail-reg.gov.uk/upload/pdf/pr08-itslicb-301008_20081117141529.pdf

We are aware of this issue, as is UIC. However, discussions with NR, UIC, and their consultants led ORR to the conclusion that any widening of the variable set in the nationwide UIC/LICB database is unlikely in the short run, although it is part of UIC's plans for the medium term. For this reason, in this update (2010) we do not assume any additional variable in the dataset although we might be able to include a larger set of (possibly firm-specific) effects in future updates of this analysis. Even in the absence of additional variables, in subsequent work we will estimate models that seek to separate unobserved heterogeneity from inefficiency, though it is debatable whether these models are able to achieve this separation in practice.

If we make the statistical assumption that any possibly omitted, firm (country)-specific effects are not correlated with the existing explanatory factors in the cost regression (as assumed in random effects, panel data MLE "stochastic frontier" efficiency estimation models of the type already used and described in PR08), then our current data-constrained approach should still give us unbiased[5] parameter estimates for the cost drivers[6].

We present below the statistical estimation results, including diagnostics, of the first maintenance and renewal (total) cost model as estimated by standard OLS (TL cost specification[7]).

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[5] This means that if repeated samples were taken the average value of each of the parameter estimates would equal their true, population values.

[6] However, if the errors are not identically and independently distributed the estimates are not efficient. Further, the estimates of the standard errors will be biased.

[7] The translog cost function is one of the so-called flexible functional forms that provides a second-order approximation to any twice differentiable cost function. It places no a priori restrictions on the input elasticities of substitution and allows the cost elasticites (and thus scale and density economies) to vary across different levels of the cost drivers.

Model 1A: Full Translog (OLS)

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------------------------------------------------------------------
Ordinary     least squares regression       LHS=TOTSS2
Fit          R-squared            =         .99267
--------+---------------------------------------------------------
        |                 Standard           Prob.       Mean
LHS Var.| Coefficient        Error      t    t>|T|       of X
--------+---------------------------------------------------------
Constant|   6.48787***       .15596     41.60 .0000
   ROUTE|   1.20433***       .13240      9.10 .0000    -.42492
  PASSDR|    .19388          .38797       .50 .6183    -.20802
    FRDR|   -.20456          .18210     -1.12 .2637    -.15786
    SING|  -1.32199***       .43738     -3.02 .0031    -.09205
    ELEC|    .34403          .25667      1.34 .1828    -.20318
    STAT|   -.20318          .34219      -.59 .5539    -.14150
  SWITCH|   -.30563          .40482      -.75 .4518    -.10388
    TIME|    .03083*         .01779      1.73 .0859    7.00000
   TIME2|   -.00097          .00118      -.82 .4116    63.0000
  ROUTE2|    .10869          .14912       .73 .4676     .94509
    PDR2|   -.29286          .48702      -.60 .5488     .50224
    FDR2|    .20987          .16030      1.31 .1931     .35932
   SING2|   -.83074          .57043     -1.46 .1481     .21106
   ELEC2|   -.41939***       .14211     -2.95 .0039     .75726
   STAT2|   1.14506***       .37771      3.03 .0030     .34025
 SWITCH2|   -.14015          .70892      -.20 .8436     .22857
   RTPDR|    .54770*         .32927      1.66 .0990     .08783
   RTFDR|    .00281          .16154       .02 .9862     .20742
  RTSING|    .21601          .32826       .66 .5119     .02058
  RTELEC|    .30862          .21898      1.41 .1615     .31324
  RTSTAT|  -1.18360***       .39961     -2.96 .0037     .07562
  RTSWIT|   -.22505          .24605      -.91 .3623     .08026
  RTTIME|   -.00294          .00823      -.36 .7216   -2.98731
 PDRSING|  -1.27094**        .63046     -2.02 .0462    -.22189
 PDRELEC|    .90079**        .35090      2.57 .0116     .30158
 PDRSTAT|   -.72701          .55974     -1.30 .1967     .33265
 PDRSWIT|    .57375          .84907       .68 .5006     .29018
 PDRTIME|    .00779          .02697       .29 .7732   -1.18695
 FDRSING|    .44299          .43758      1.01 .3135    -.06102
 FDRELEC|   -.19474          .16229     -1.20 .2327     .34937
 FDRSTAT|   -.62421*         .34910     -1.79 .0765     .21713
 FDRSWIT|    .60893          .56758      1.07 .2856     .19676
 FDRTIME|    .01063          .01453       .73 .4658   -1.17207
 SNGELEC|   1.28860**        .54725      2.35 .0203    -.07910
 SNGSTAT|    .59192          .64712       .91 .3623    -.13759
 SNGSWIT|   -.81443         1.10563      -.74 .4629    -.13845
 SNGTIME|   -.00297          .02251      -.13 .8953    -.72490
 ELESTAT|   1.51065***       .55137      2.74 .0072     .21197
 ELESWIT|   -.41424          .54098      -.77 .4454     .25040
 ELETIME|   -.02632**        .01163     -2.26 .0255   -1.33623
 STASWIT|  -1.56179**        .72798     -2.15 .0341     .21403
 STATIME|   -.01481          .01791      -.83 .4101    -.97010
 SWITIME|    .02593          .03859       .67 .5030    -.82782
--------+---------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
------------------------------------------------------------------
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This overall translog model (Model 1A) produces implausible sizes and signs of the parameters (elasticities[8]) on the main passenger and train density variables at the sample mean, and most of the

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[8] Since the cost equation in a TL model is expressed in logs and the right hand side variables are normalised at the sample mean, the estimated parameters on the first order network output/scale and train density variables can be directly interpreted as elasticities in an economic sense. A cost elasticity gives the percentage variation, other things being equal (partial derivative), of the dependent variable in the equation (in our case, cost) corresponding to a 1% variation in the independent variable (for instance, scale or density).

first order terms are statistically insignificant at conventional confidence levels, implying that the cost elasticities with respect to these variables are insignificant at the sample mean. This is a common finding with translog specifications, given the complexity of the functional form when there are large numbers of explanatory variables in combination with a relatively small cross section. A restricted translog model was therefore tried, which only includes second order variables for what might be considered as the main network output (scale and density) variables, namely route-km and passenger and freight train density. The econometric results of this restricted model (Model 1A1) are shown below, estimated once again by standard OLS.

The estimation results are again unsatisfactory. At the sample mean, the passenger elasticity is high compared to previous studies and the freight elasticity is negative (though statistically insignificant), which is counter-intuitive. The negative and significant coefficient on the proportion of electrified track might also be considered counter-intuitive. Looking at the cost elasticities away from the sample mean, we also see implausibly large negative cost elasticities. Thus we do not proceed with the translog models, although we note that for the OLS models, the Cobb-Douglas restriction is rejected.

In the subsequent sections we proceed using stochastic frontier models applied initially to the Cobb-Douglas cost frontier. We then double-check by adding second-order terms back in later and compute appropriate hypothesis tests.

Model 1A1: Restricted Translog 1 (OLS)

```
-----------------------------------------------------------------
Ordinary     least squares regression LHS=TOTSS2
Fit          R-squared              =        .97736
             Adjusted R-squared     =        .97511
Diagnostic   Log likelihood         =      58.37609
--------+--------------------------------------------------------
        |                Standard            Prob.       Mean
LHS Var.| Coefficient      Error        t    t>|T|       of X
--------+--------------------------------------------------------
Constant|     6.84784***      .08889    77.03  .0000
   ROUTE|     1.11422***      .02462    45.25  .0000     -.42492
  PASSDR|      .55501***      .13322     4.17  .0001     -.20802
    FRDR|     -.05530         .08184     -.68  .5004     -.15786
    SING|     -.54326***      .07868    -6.91  .0000     -.09205
    ELEC|     -.28137***      .06055    -4.65  .0000     -.20318
    STAT|      .46551***      .10745     4.33  .0000     -.14150
  SWITCH|      .32511**       .12463     2.61  .0101     -.10388
    TIME|      .03833**       .01651     2.32  .0217     7.00000
   TIME2|     -.00212*        .00115    -1.84  .0680     63.0000
  ROUTE2|     -.28216***      .05359    -5.26  .0000      .94509
    PDR2|      .41565***      .06174     6.73  .0000      .50224
    FDR2|      .01632         .04315      .38  .7059      .35932
   RTPDR|      .06233         .10228      .61  .5432      .08783
   RTFDR|      .02869         .07348      .39  .6968      .20742
--------+--------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------
```

3. The Cobb-Douglas (CD) maintenance and renewal cost model in Cuesta (2000)

We now consider the appropriate model specification, assuming a Cobb-Douglas functional form. We use the Cuesta (2000) model. This models firm-specific cost inefficiency in a panel data setting (a time series of cross sections); as a function of time t, in the form[9]:

$$u_{it} = u_i \cdot \exp(\eta_i \cdot (T - t)) \quad (1)$$

where

u_{it} = time-varying inefficiency (distance from best-practice stochastic cost frontier) of firm i at time point t;

η_i = a firm-specific parameter (eta) to be estimated;

t = time (in years) and T is the last year of the panel; and

u_i = time-invariant (constant) inefficiency level for firm i.

The estimation results for this model are shown below.

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[9] Details are provided in our PR08 published international benchmarking documentation.

Model 1B: Cobb-Douglas/CD [Cuesta (2000): $U_{it} = \exp(\eta_i.t).U_i)$][10]

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-------------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable              TOTSS2
Log likelihood function       98.74401
Estimation based on N =    156, K =  26
--------+----------------------------------------------------------
        |                Standard            Prob.        Mean
LHS Var.| Coefficient      Error       z    z>|Z|        of X
--------+----------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.37037***      .07210    88.35  .0000
   ROUTE|     .96551***      .04755    20.31  .0000     -.42492
  PASSDR|    -.19044         .18627    -1.02  .3066     -.20802
    FRDR|     .02478         .10104      .25  .8062     -.15786
    SING|    -.54430***      .07871    -6.92  .0000     -.09205
    ELEC|     .24171**       .11014     2.19  .0282     -.20318
    STAT|     .62291***      .10410     5.98  .0000     -.14150
  SWITCH|     .28266         .18047     1.57  .1173     -.10388
    TIME|     .05833***      .01181     4.94  .0000     7.00000
   TIME2|    -.00527***      .00115    -4.57  .0000     63.0000
        |Variance parameters for compound error
  Lambda|    7.25813***      .03722   195.00  .0000
Sigma(u)|     .78616***      .12938     6.08  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|
    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|     .02885         .05071      .57  .5695
    XT13|     .05478         .05405     1.01  .3108
  XT13SQ|    -.01251*        .00670    -1.87  .0618
--------+----------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
-------------------------------------------------------------------
```

Including all variables together in the CD model specification (Model 1B) results in the passenger train density and overall cost elasticity being negative (albeit statistically insignificant), which is not in line with theoretical expectations. We thus proceed to drop statistically insignificant explanatory variables, dropping the ones showing the lowest t-statistic (in absolute value) first. This leads us to three gradually refined CD models, which are shown below.

Model 1B1: Cobb-Douglas [Cuesta (2000): $U_{it} = \exp(\eta_i.(T-t).U_i))$], restricted

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-------------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable              TOTSS2
Log likelihood function       95.64474
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[10] All of the models presented in this publication were estimated using the LIMDEP statistical/econometric package (William Greene/Econometric Software Inc.), version 9.

```
Estimation based on N =    156, K =  23
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC               -.93134     -145.28947
Fin.Smpl.AIC      -.87773     -136.92583
Bayes IC          -.48168      -75.14278
Hannan Quinn      -.74871     -116.79892
--------+---------------------------------------------------------
        |                  Standard           Prob.       Mean
LHS Var.| Coefficient        Error       z    z>|Z|       of X
--------+---------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.35070***      .05636   112.68  .0000
   ROUTE|    1.06169***      .02509    42.31  .0000     -.42492
    SING|    -.83236***      .07515   -11.08  .0000     -.09205
    STAT|     .26251*        .14846     1.77  .0770     -.14150
  SWITCH|     .21022*        .11972     1.76  .0791     -.10388
    TIME|     .04806***      .01148     4.18  .0000     7.00000
   TIME2|    -.00375***      .00083    -4.50  .0000     63.0000
        |Variance parameters for compound error
  Lambda|    4.84765***      .04658   104.07  .0000
Sigma(u)|     .54673***      .03841    14.24  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|
    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|     .02279         .07856      .29  .7718
    XT13|     .12654         .08582     1.47  .1404
  XT13SQ|    -.02194**       .01055    -2.08  .0376
--------+---------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
------------------------------------------------------------------
```

Model 1B2: Cobb-Douglas [(Cuesta (2000): U_{it} = exp(η_i.(T-t)).U_i)], restricted (retaining passenger and freight densities)

```
------------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable               TOTSS2
Log likelihood function        97.49756
Estimation based on N =    156, K =  25
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC               -.92946     -144.99513
Fin.Smpl.AIC      -.86535     -134.99513
Bayes IC          -.44070      -68.74873
Hannan Quinn      -.73094     -114.02714
--------+---------------------------------------------------------
        |                  Standard           Prob.       Mean
LHS Var.| Coefficient        Error       z    z>|Z|       of X
--------+---------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.45439***      .04423   145.92  .0000
   ROUTE|    1.01603***      .03475    29.24  .0000     -.42492
  PASSDR|     .02407         .09722      .25  .8044     -.20802
    FRDR|     .04989         .08998      .55  .5793     -.15786
```

```
    SING|    -.60100***      .06859    -8.76  .0000    -.09205
    ELEC|     .21283*        .11008     1.93  .0532    -.20318
    STAT|     .64029***      .09156     6.99  .0000    -.14150
    TIME|     .05544***      .01166     4.75  .0000    7.00000
   TIME2|    -.00507***      .00106    -4.77  .0000    63.0000
        |Variance parameters for compound error
  Lambda|    6.27121***      .04000   156.79  .0000
Sigma(u)|     .69218***      .08316     8.32  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|
    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|     .02891         .06655      .43  .6640
    XT13|     .07185         .06237     1.15  .2493
  XT13SQ|    -.01564**       .00763    -2.05  .0403
--------+---------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
------------------------------------------------------------------
```

Model 1B3: Cobb-Douglas [Cuesta (2000): U_{it} = exp(η_i.(T-t)).U_i)], restricted (excluding station density)

```
-----------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable               TOTSS2
Log likelihood function        92.12535
Estimation based on N =    156, K =  23
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC               -.88622     -138.25069
Fin.Smpl.AIC      -.83261     -129.88706
Bayes IC          -.43656      -68.10400
Hannan Quinn      -.70359     -109.76014
--------+--------------------------------------------------------
        |                Standard          Prob.       Mean
LHS Var.| Coefficient      Error       z    z>|Z|       of X
--------+--------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.39762***      .04711   135.79  .0000
   ROUTE|    1.12216***      .03592    31.24  .0000    -.42492
  PASSDR|     .30390***      .06527     4.66  .0000    -.20802
    FRDR|    -.00757         .10494     -.07  .9425    -.15786
    SING|   -1.00598***      .09384   -10.72  .0000    -.09205
    TIME|     .04266***      .01158     3.68  .0002    7.00000
   TIME2|    -.00394***      .00087    -4.54  .0000    63.0000
        |Variance parameters for compound error
  Lambda|    4.92141***      .04667   105.45  .0000
Sigma(u)|     .57339***      .04473    12.82  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|
    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|    -.10538         .15031     -.70  .4832
    XT13|     .15107         .14289     1.06  .2904
  XT13SQ|    -.03061         .01876    -1.63  .1027
--------+--------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------
```

The disadvantage of Model 1B1 above is that, although the remaining explanatory variables are statistically significant, it does not include any volume (output) measures in the final specification. We therefore test down from the full Cobb-Douglas specification, but always preferring to keep the passenger and freight train density variables (as a minimum) in the model. This produces Model 1B2. However, both the passenger and freight train density variables are still close to zero and insignificant in this model in explaining cost, which is not in line with expectations. It could be that the high standard errors are the result of multi-collinearity[11], thus creating a situation in which there is considerable uncertainty surrounding the true importance of the individual explanatory variables. We therefore go a step further and drop the station density variable, which leaves us with Model 1B3. In this model, the passenger train densities coefficient is now statistically significant. The freight elasticity is approximately zero and insignificant which again could be the result of multi-collinearity. However, the total elasticity of cost with respect to all traffic is within estimates from previous studies. In section 6 we consider combining passenger and train density into a single variable.

Overall, from a theoretical and presentational perspective there is an argument for preferring a cost model that contains a measure of volume (output) in place of other variables (such as station density) which might be acting as a proxy for traffic/throughput on the network, although this point would warrant further discussion. The alternative argument is that, in fact, it is better to leave significant variables in the model, and accept some lack of precision on individual parameter estimates that may occur due to multi-collinearity. We note that out of models 1B and its variants 1B1 to 1B3, the selected model (model 1B3) under the Cuesta (2000) specification gives the highest cost efficiency score for Network Rail (the scores for Models 1B, 1B1, 1B2 and 1B3 are 0.30, 0.53, 0.40 and 0.66 respectively). We also note that leaving the percentage of electrified lines as a variable in the model makes little difference to the other parameter estimates and/or the cost efficiency scores. As in PR08, the coefficient on the electrification variable is small and statistically insignificant, and this variable was therefore dropped from the preferred model.

4. Further tests on the preferred model – different specification of the Cuesta (2000) model

As noted above, following Cuesta (2000) we adopt the following cost inefficiency specification.

$$U_{it} = \exp(\eta_i.(T-t)).U_i \qquad (2)$$

Alternative variants around this standard functional form are possible. Preliminary work has suggested that the results for Network Rail are not very sensitive to small changes to the specification(the main variant tried to date produces a score of 0.60 for Network Rail in 2008; Model 1B4; full results not shown).This is an area for further investigation in the next stage of work.

5. Further tests on the preferred model – checking against Translog alternatives

As noted above, preliminary estimation and testing using standard OLS techniques suggested that the Cobb-Douglas restriction should be rejected, though the translog parameter estimates appeared implausible. Having selected and then worked with a Cobb-Douglas functional form in a stochastic frontier/MLE panel data context (Model 1B3), for completeness we look at whether this can be broadened out back into a restricted translog based on the stochastic frontier Cuesta/MLE (2000), rather than a simple OLS, modelling approach. However, we already noted that a full-variable

---

[11] Multi-collinearity may occur when one or more of the right-hand side variables in the estimated equation are highly correlated with each other. Multi-collinearity often manifests itself in high standard errors.

translog Cuesta model failed to converge, so we now include second order terms only for the key output / scale variables: route-km (output/volume) and passenger and freight densities (Model 1C).

Model 1C: Restricted TL [(Cuesta (2000): $U_{it} = \exp(\eta_i.(T-t)).U_i$)]

```
------------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable               TOTSS2
Log likelihood function        97.62361
Estimation based on N =    156, K =  29
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC              -.87979     -137.24722
Fin.Smpl.AIC     -.79127     -123.43769
Bayes IC         -.31283      -48.80139
Hannan Quinn     -.64952     -101.32435
--------+---------------------------------------------------------
        |                  Standard           Prob.       Mean
LHS Var.| Coefficient        Error       z    z>|Z|       of X
--------+---------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.44416***      .14740    43.72  .0000
   ROUTE|    1.15257***      .02712    42.49  .0000    -.42492
  PASSDR|     .43110***      .15299     2.82  .0048    -.20802
    FRDR|    -.16826         .10592    -1.59  .1122    -.15786
    SING|    -.97382***      .12367    -7.87  .0000    -.09205
    TIME|     .03658***      .01273     2.87  .0040    7.00000
   TIME2|    -.00358***      .00086    -4.14  .0000    63.0000
  ROUTE2|     .02706         .07297      .37  .7107     .94509
    PDR2|    -.01280         .17417     -.07  .9414     .50224
    FDR2|    -.09762         .08891    -1.10  .2722     .35932
   RTPDR|     .12833         .09564     1.34  .1796     .08783
   RTFDR|    -.15120         .13565    -1.11  .2650     .20742
    PRDR|     .29966         .19284     1.55  .1202     .19971
        |Variance parameters for compound error
  Lambda|    5.46741***      .04464   122.49  .0000
Sigma(u)|     .61122***      .05774    10.59  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|

    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|   -4.88436       68607.59      .00  .9999
    XT13|     .20394         .21542      .95  .3438
  XT13SQ|    -.04325         .02773    -1.56  .1188
--------+---------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
------------------------------------------------------------------
```

An LR test on Model 1C reveals that we cannot reject the null hypothesis of the Cobb-Douglas restriction (including a time-trend squared term) versus the translog alternative (TL); thus providing support for the preferred model. It should be noted that the TL model implies an efficiency score of 0.76 for Network Rail. However, again this model has a negative freight elasticity at the sample mean that is close to being statistically significant (p value of 0.11), and many of the elasticities away from the sample mean are therefore negative. Taking all of these factors into account, we therefore prefer the Cobb-Douglas specification in 1B3; this is a finding that is also in line with the OLS analysis carried out previously, as well as with the PR08 efficiency econometrics. We therefore accept the restriction on the TL and proceed with the CD cost function specification.

6. Further tests on the preferred model – checking against total train density models

Below we adapt the preferred model by replacing the passenger and freight density variables with total train density (Model 1D). This model produces sensible elasticities and produces an efficiency score of 0.63 for Network Rail in 2008, very similar to that of the preferred model. It therefore provides further support for the preferred model. The translog equivalent produces a similar score for Network Rail and the Cobb-Douglas restriction cannot be rejected.

Model 1D: Cobb Douglas Total Train Density Model [(Cuesta (2000): $U_{it} = \exp(\eta_i.(T-t)).U_i$)]

```
------------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable               TOTSS2
Log likelihood function        91.09969
Estimation based on N =    156, K =  22
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC              -.88589     -138.19938
Fin.Smpl.AIC     -.83712     -130.59036
Bayes IC         -.45579      -71.10255
Hannan Quinn     -.71120     -110.94755
--------+---------------------------------------------------------
        |                 Standard          Prob.       Mean
LHS Var.| Coefficient       Error       z    z>|Z|       of X
--------+---------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.38894***      .04596   139.00  .0000
   ROUTE|    1.10513***      .02692    41.05  .0000     -.42492
   TOTDR|     .37573***      .08653     4.34  .0000     -.16462
    SING|    -.97848***      .07001   -13.98  .0000     -.09205
    TIME|     .04334***      .01192     3.63  .0003     7.00000
   TIME2|    -.00396***      .00089    -4.48  .0000     63.0000
        |Variance parameters for compound error
  Lambda|    4.82954***      .04656   103.72  .0000
Sigma(u)|     .56570***      .04199    13.47  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|

    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|    -.05345         .10469     -.51  .6096
    XT13|     .13540         .12255     1.10  .2693
  XT13SQ|    -.02695*        .01498    -1.80  .0720
--------+---------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
------------------------------------------------------------------
```

7. Further tests on the preferred model – checking against models containing track-km (as opposed to route-km) as an output variable

We now test the preferred model against a different scale (output) variable: track-km instead of route-km. Track-km is a "circuit" variable in that it sums up the length of all tracks along the physical route. Therefore, it is a multiple of route-km in the presence of more than one track on the line, and coincides with route-km if there is just one track on the line. The group of models including track-km is termed "Model 2". Model 2 is specified as follows (all variables are in natural logs):

- Track-km (TRACK);
- % of single track (SING);
- % of electrified track (ELEC);
- Passenger train density per track-km (PASSDT);
- Freight train density per track-km (FRDT);
- Or total train density per track-km (TOTDT);
- Station density per route-km[12] (STAT); and
- Switch density per track-km (SWITCH).

As before, testing down the model produces two variants (Models 2A and 2B), depending on whether we decide to retain the passenger and train density variables irrespective of their significance, or drop them when insignificant. Model 2B, which is identical to the preferred model, Model 1B3 (except for being denominated in track-km now), gives an efficiency score for Network Rail of 0.68, which is very close to the result coming out of the preferred model. Generally Model 2B is very similar to the preferred model, though the latter has a higher log-likelihood value. We therefore continue with model 1B3 as a preferred model. We reached a similar conclusion in the PR08 modelling.

We also note that the restricted translog version (Model 2B1; model output not shown) of these models produces similar results to those of the route-km models (again, the Cobb-Douglas restriction cannot be rejected at the 5% significance level). Once again, however, the cost elasticities do not seem believable, particularly as we move away from the sample mean. For numerous observations, and Network Rail in particular, the cost elasticity with respect to freight train density becomes negative.

---

[12] Station density is still per route-km as stations are a function of passenger traffic and route length, not of the number of tracks along the line.

Model 2A: Cobb Douglas [(Cuesta (2000): $U_{it} = \exp(\eta_i.(T-t)).U_i$)], restricted

```
------------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable              TOTSS2
Log likelihood function        91.83262
Estimation based on N =    156, K =  23
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC              -.88247     -137.66523
Fin.Smpl.AIC     -.82886     -129.30160
Bayes IC         -.43281      -67.51855
Hannan Quinn     -.69984     -109.17469
--------+---------------------------------------------------------
        |                 Standard          Prob.       Mean
LHS Var.| Coefficient       Error       z    z>|Z|       of X
--------+---------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.40728***      .04858   131.89  .0000
   TRACK|     .95856***      .06125    15.65  .0000     -.48446
    SING|    -.45981***      .11052    -4.16  .0000     -.09205
    ELEC|     .27386*        .15285     1.79  .0732     -.20318
  SWITCH|     .48937***      .10421     4.70  .0000     -.10388
    TIME|     .04027***      .01252     3.22  .0013     7.00000
   TIME2|    -.00378***      .00095    -3.96  .0001     63.0000
        |Variance parameters for compound error
  Lambda|    6.54824***      .04245   154.25  .0000
Sigma(u)|     .74172***      .11035     6.72  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|

    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|     .01832         .05646      .32  .7456
    XT13|     .08243         .06252     1.32  .1874
  XT13SQ|    -.01528**       .00774    -1.97  .0484
--------+---------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
------------------------------------------------------------------
```

Model 2B: Cobb Douglas [Cuesta (2000): U_{it} = exp(η_i.(T-t)).U_i)], restricted

```
------------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable               TOTSS2
Log likelihood function        89.44737
Estimation based on N =    156, K =  23
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC              -.85189     -132.89474
Fin.Smpl.AIC     -.79828     -124.53111
Bayes IC         -.40223      -62.74805
Hannan Quinn     -.66926     -104.40419
--------+---------------------------------------------------------
        |                  Standard           Prob.       Mean
LHS Var.| Coefficient        Error       z    z>|Z|       of X
--------+---------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.50005***      .04508   144.18  .0000
   TRACK|    1.09401***      .02991    36.58  .0000    -.48446
  PASSDT|     .30726***      .06159     4.99  .0000    -.14091
    FRDT|     .07566         .09461      .80  .4239    -.14630
    SING|    -.76452***      .05596   -13.66  .0000    -.09205
    TIME|     .03550***      .01202     2.95  .0031    7.00000
   TIME2|    -.00340***      .00088    -3.86  .0001    63.0000
        |Variance parameters for compound error
  Lambda|    4.58136***      .04937    92.80  .0000
Sigma(u)|     .54678***      .03924    13.93  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|
        |
    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|    -.12134         .15015     -.81  .4190
    XT13|     .17415         .15707     1.11  .2675
  XT13SQ|    -.03237         .02014    -1.61  .1079
--------+---------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
------------------------------------------------------------------
```

In what follows, we move back to the preferred model based on main network output expressed as route-km, and proceed with further sensitivity testing.

8. Further tests on the preferred (route-km) model – single track percentage, electrification percentage, station and switch densities now in natural values

Testing down the Cobb-Douglas version with percentage or ratio variables not in natural logs (i.e., left as natural values) produces two variants (Models 2C and 2D), depending on whether we decide to retain the passenger and train density variables irrespective of their significance, or drop them when insignificant. As before, in order to arrive at model 2D, we have to take out significant variables in order to obtain positive coefficients on the key output variables. Overall, there is nothing in these results to suggest a change to our choice of preferred model. The parameter estimates are less plausible on the key output variables. We note that Network Rail's efficiency score in 2008 is lower than for the preferred (fully logarithmic) model.

Model 2C: Cobb Douglas [(Cuesta (2000): U_{it} = exp(η_i.(T-t)).U_i)], restricted and partially "un-logged" (ratio variables)

```
------------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable              TOTSS2
Log likelihood function       100.77694
--------+---------------------------------------------------------
        |                Standard           Prob.        Mean
LHS Var.| Coefficient      Error       z    z>|Z|        of X
--------+---------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.65048***      .11924    55.78  .0000
   ROUTE|    1.03908***      .01924    54.00  .0000    -.42492
   SINGB|    -.97430***      .05612   -17.36  .0000    1.00000
   ELECB|     .13333**       .06774     1.97  .0490    1.00000
   STATB|     .59638***      .04037    14.77  .0000    1.00000
    TIME|     .05812***      .01184     4.91  .0000    7.00000
   TIME2|    -.00439***      .00093    -4.74  .0000    63.0000
        |Variance parameters for compound error
  Lambda|    4.27743***      .05434    78.71  .0000
Sigma(u)|     .47638***      .02648    17.99  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|

    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|     .03337         .07387      .45  .6514
    XT13|     .13001         .08369     1.55  .1203
  XT13SQ|    -.02203**       .01014    -2.17  .0298
--------+---------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
------------------------------------------------------------------
```

Model 2D: Cobb Douglas [(Cuesta (2000): U_{it} = exp(η_i.(T-t)).U_i)], restricted and partially "un-logged" (ratio variables)

```
------------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
```

```
Dependent variable               TOTSS2
Log likelihood function        93.92176
Estimation based on N =    156, K =  23
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC              -.90925     -141.84351
Fin.Smpl.AIC     -.85564     -133.47987
Bayes IC         -.45960      -71.69682
Hannan Quinn     -.72662     -113.35296
--------+------------------------------------------------------------
        |                Standard            Prob.        Mean
LHS Var.| Coefficient      Error       z    z>|Z|        of X
--------+------------------------------------------------------------
        |Primary Index Equation for Model
Constant|    7.83582***      .10766    72.78  .0000
   ROUTE|    1.07417***      .02744    39.15  .0000     -.42492
  PASSDR|     .05303         .07097      .75  .4550     -.20802
    FRDR|     .02649         .08388      .32  .7522     -.15786
   SINGB|   -1.44699***      .11370   -12.73  .0000     1.00000
    TIME|     .04610***      .01159     3.98  .0001     7.00000
   TIME2|    -.00349***      .00082    -4.25  .0000     63.0000
        |Variance parameters for compound error
  Lambda|    4.34076***      .05098    85.14  .0000
Sigma(u)|     .50118***      .02936    17.07  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|
    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|    -.03600         .10171     -.35  .7234
    XT13|     .17561         .12098     1.45  .1466
  XT13SQ|    -.03044**       .01503    -2.03  .0428
--------+------------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
---------------------------------------------------------------------
```

9. Further tests on the preferred model: time-invariant inefficiency scores and other models

Below, we show the Battese and Coelli (1992)[13] and Pitt and Lee (1981)[14] models which are both nested in our preferred model (they are special cases of the more general model). These are Models 3A and 3B below. Respectively these models assume the direction of inefficiency change to be the same for all firms or that inefficiency is time invariant for all firms. We consider that both of these assumptions are unrealistic, especially given the experience of Network Rail and its predecessor Railtrack.

Likelihood Ratio (LR) tests show that both time-constrained restrictions on the general, time-varying inefficiency error specification can be rejected. For the Battese and Coelli (1992) model, the LR ratio

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[13] Battese, G.E., and Coelli, T.J. (1992): “Frontier Production Functions, Technical Efficiency and Panel Data: With Application to Paddy Farmers in India”, Journal of Productivity Analysis, vol. 3, pp. 153-169.

[14] Pitt, M.M. and Lee, L.F. (1981), ‘Measurement and Sources of Technical Inefficiency in the Indonesian Weaving Industry’, Journal of Development Economics, 9,43-64.

test statistic is (92.12-53.96)*2 =76.32. The 5% critical value (13 restrictions) is 22.36, so we strongly reject the restriction of the same direction of inefficiency change for all firms. Likewise, the LR ratio test statistic for the simpler Pitt and Lee (1981) model restriction is (92.12-53.33)*2 = 77.58. The 5% critical value (14 restrictions) in this case is 23.68, so once again this is a clear rejection of time invariance in inefficiency scores. More specifically, we note that in our preferred model (Model 1B3) the time variation in inefficiency for Network Rail is statistically significant at the 5% level, indicating our preference for a model that permits the company's inefficiency to change over time.

As with our experience during PR08 and in other (academic) contexts, the Battese and Coelli (1992) model does not produce very sensible results in respect of the parameter estimates. This model shows the efficiency of all firms deteriorating over time (one-directionality) and gives a score for Network Rail in 2008 of just 0.48. On the other hand, the time invariant model (Pitt and Lee, 1981) produces more sensible results (again, as found in PR08 and in other modelling work), giving a score for Network Rail in 2008 of 0.60, more in line with the preferred (time varying) models (Model 1B3).

Model 3A: Cobb-Douglas cost function (Battese and Coelli, 1992) restricted, with time-varying inefficiency in the same direction for all firms

```
-----------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable               TOTSS2
Log likelihood function        53.96022
Estimation based on N =    156, K =  10
Information Criteria: Normalization=1/N
             Normalized   Unnormalized
AIC             -.56359      -87.92043
Fin.Smpl.AIC    -.55387      -86.40319
Bayes IC        -.36809      -57.42187
Hannan Quinn    -.48419      -75.53324
--------+--------------------------------------------------------
        |                 Standard            Prob.       Mean
LHS Var.| Coefficient       Error       z    z>|Z|       of X
--------+--------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.16511***      .14209    43.39  .0000
   ROUTE|     .97437***      .11034     8.83  .0000    -.42492
  PASSDR|    -.02726         .18573     -.15  .8833    -.20802
    FRDR|    -.09947         .07841    -1.27  .2046    -.15786
    SING|   -1.10243***      .18979    -5.81  .0000    -.09205
    TIME|     .02849**       .01401     2.03  .0420    7.00000
   TIME2|    -.00206**       .00096    -2.15  .0317    63.0000
        |Variance parameters for compound error
  Lambda|    4.01929***      .06922    58.07  .0000
Sigma(u)|     .58849***      .06062     9.71  .0000
        |Eta parameter for time varying inefficiency
     Eta|    -.02065*        .01148    -1.80  .0720
--------+--------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------
```

Model 3B: Cobb-Douglas cost function (Pitt and Lee, 1981), restricted –time-invariant efficiency model

```
-----------------------------------------------------------------
Limited Dependent Variable Model - FRONTIER
Dependent variable              TOTSS2
Log likelihood function       53.33546
Estimation based on N =    156, K =   9
Information Criteria: Normalization=1/N
             Normalized   Unnormalized
AIC             -.56840      -88.67092
Fin.Smpl.AIC    -.56050      -87.43804
Bayes IC        -.39245      -61.22221
Hannan Quinn    -.49694      -77.52244

--------+--------------------------------------------------------
        |                 Standard           Prob.        Mean
LHS Var.| Coefficient       Error       z    z>|Z|        of X
--------+--------------------------------------------------------
        |Primary Index Equation for Model
Constant|    6.29523***      .13151    47.87  .0000
   ROUTE|    1.03393***      .09594    10.78  .0000    -.42492
  PASSDR|     .21118         .14424     1.46  .1432    -.20802
    FRDR|    -.04715         .06343     -.74  .4572    -.15786
    SING|    -.97302***      .11980    -8.12  .0000    -.09205
    TIME|     .03098**       .01430     2.17  .0302    7.00000
   TIME2|    -.00188*        .00097    -1.94  .0525    63.0000
        |Variance parameters for compound error
  Lambda|    2.85848**      1.22080     2.34  .0192
Sigma(u)|     .42762***      .10300     4.15  .0000
--------+--------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------
```

10. Sensitivity Analysis on the Steady-State Adjustment and Corrected Ordinary Least Squares (COLS) as a back-up to the preferred model

Before concluding, we also estimate a total cost model whereby monetary values are still adjusted by PPPs and GDP-deflated, but without a steady-state adjustment (long run equilibrium renewal rate) for Network Rail. This sensitivity results from post-PR08 debates that ORR has had with stakeholders (as well as internally) on the appropriateness of extending the renewals' steady-state adjustment for Network Rail into the next control period. This model produces similar parameter estimates to the preferred model. This model variant (Model 1E) gives a score of just 0.53 for NR in 2008, which seems plausible (at least directionally), given that renewal costs post-Hatfield are not being manually reduced here as opposed to the "steady-state" adjusted cost data used in the remainder of this analysis and in PR08.

Model 1E: Cobb-Douglas [Cuesta (2000): $U_{it} = \exp(\eta_i.(T-t)).U_i)$], restricted (excluding station density) – Dependent Variable=Total Costs (unadjusted)

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Limited Dependent Variable Model - FRONTIER
Dependent variable                  TOT
Log likelihood function        93.16771
Estimation based on N =    156, K =  23
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC              -.89959     -140.33542
Fin.Smpl.AIC     -.84597     -131.97179
Bayes IC         -.44993      -70.18874
Hannan Quinn     -.71695     -111.84488
--------+---------------------------------------------------------
        |                 Standard          Prob.       Mean
LHS Var.| Coefficient       Error       z    z>|Z|       of X
--------+---------------------------------------------------------
        |Primary Index Equation for Model
Constant|     6.39068***      .04969   128.62  .0000
   ROUTE|     1.10157***      .02521    43.70  .0000    -.42492
  PASSDR|      .30427***      .06999     4.35  .0000    -.20802
    FRDR|      .05645         .06049      .93  .3507    -.15786
    SING|    -.96248***       .06896   -13.96  .0000    -.09205
    TIME|      .04457***      .01172     3.80  .0001    7.00000
   TIME2|    -.00399***       .00087    -4.59  .0000    63.0000
        |Variance parameters for compound error
  Lambda|     5.10616***      .04739   107.75  .0000
Sigma(u)|      .59144***      .05132    11.52  .0000
        |Coefficients in u(i,t)=[exp{eta*z(i,t)}]*|U(i)|

    XT01|    Results excluded to protect confidentiality
    XT02|    of non-British firms
    XT03|
    XT04|
    XT05|
    XT06|
    XT07|
    XT08|
    XT09|
    XT10|
    XT11|
    XT12|   -4.05025       69255.08       .00 1.0000
    XT13|     .06860         .09404       .73  .4657
  XT13SQ|    -.01692         .01179     -1.44  .1512
--------+---------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
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```

Finally, we note that a simpler, Corrected Ordinary Least Squares (COLS) model, as discussed in PR08 and pertaining documentation, gives a score for NR in 2008 of 0.65 against the upper quartile (the best 25% of observed firms). This model also produces similar rankings to the preferred stochastic frontier (MLE) model with respect to the frontier firms (this ranking is confidential due to our agreements with UIC/LICB).

The profile of the COLS scores for Railtrack (RT-GB) and Network Rail (NR-GB) over time is shown below. It seems broadly compatible with the stochastic frontier model, although inefficiency variation over time appears to be more limited in COLS for Network Rail (especially in the later years).

12	United Kingdom - RT	1996	0.684901
12	United Kingdom - RT	1997	0.701403
12	United Kingdom - RT	1998	0.745157
12	United Kingdom - RT	1999	0.80895
13	United Kingdom - NR	2000	0.816223
13	United Kingdom - NR	2001	0.656485
13	United Kingdom - NR	2002	0.605582
13	United Kingdom - NR	2003	0.552783
13	United Kingdom - NR	2004	0.523354
13	United Kingdom - NR	2005	0.662332
13	United Kingdom - NR	2006	0.647541
13	United Kingdom - NR	2007	0.645823
13	United Kingdom - NR	2008	0.647957

11. Conclusions

Our preferred inefficiency model is shown to be robust compared to various tests against numerous alternatives in terms of the choice of variables, functional form and efficiency model specification. The preferred model implies a score for Network Rail in 2008 of 0.66 (efficiency gap of 34%). The comparator models in general produce lower scores. Overall, at this stage, we consider that the score of 0.66 for Network Rail in 2008 is a conservative estimate.

One possible further area for exploration is to discuss with railway engineers the implications of the statistical coefficient sizes and signs in the preferred and other models. Some of the translog cost specifications produced smaller inefficiency gaps, although it appeared that the equation coefficients were not sensible for these cases and, in most cases, we could not reject the Cobb-Douglas restriction versus the translog. Further exploration of the railway engineering implications of the coefficients, in tandem with further work on the translog cost function (and perhaps other functional forms) would be a valuable addition to our efficiency work stream during PR13. We also expect to undertake further work in some or all of the following areas:

- the precise functional form for the time varying inefficiency specification;
- the characterisation of the time trend;
- random parameter models;
- presence of heteroscedasticity in the error term;
- semi-parametric estimation and different (cost) functional forms; and
- computation of confidence intervals for efficiency estimates.

The Appendix below provides the NR efficiency score (2008 sample time point) from all total cost models estimated as part of this efficiency econometrics update (2010).

Appendix 1: Efficiency scores from all models.

Efficiency scores and gaps for Network Rail (2008)

Model number	1B	1B1	1B2	1B3	1B4	1C*	1D	1E	1F
Efficiency score	30%	53%	41%	66%	60%	76%	63%	53%	65%
Efficiency gap	70%	47%	59%	34%	40%	24%	37%	47%	35%

Model number	2A	2B	2B1*	2C	2D	3A	3B
Efficiency score	36%	68%	80%	51%	63%	48%	60%
Efficiency gap	64%	32%	20%	49%	37%	52%	40%

* Translog models (TL). The Cobb-Douglas (CD) restriction cannot be rejected, and the TL models produce problematic cost/output and cost/density elasticities.